LITERATURE CITED

1. L. G. Kachurin and V. G. Morachevskii, Kinetics of Water Phase Transitions in the Atmosphere [in Russian], Izd. Leningr. Gos. Univ., Leningrad (1965).
2. I. P. Mazin, Physical Bases of Airplane Icing [in Russian], Gidrometeoizdat, Moscow (1957).
3. A. E. Carte, "Air bubbles in ice," Proc. Phys. Soc., 77, No. 495, 757-768 (1961).
4. M. K. Zhekamukhov, "A theory of hailstone growth in clouds," Tr. VGI, No. 19 (1971).
5. B. J. Mason, Cloud Physics [Russian translation], Gidrometeoizdat, Leningrad (1961).
6. E. A. Moelwyn-Hughes, Physical Chemistry, 2nd ed., Pergamon (1964).
7. M. A. Lavrent'ev and B. V. Shabat, Methods of the Theory of Functions of a Complex Variable [in Russian], GIFML, Moscow (1958).
8. H. Bateman and A. Erdelyi (editors), Tables of Integral Transforms, McGraw-Hill (1954).

NUMERICAL SOLUTION OF LIPPMANN – SCHWINGER INTEGRAL EQUATION

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A method is discussed for obtaining a numerical solution of an equation similar to the equation for the transport of radiant energy for a steady radiation field [1].

The integral equation for thermal radiation, taking account of scattering, has the form [2]

$$t_l(p;\, p_1;\, k^2) = V_l(p;\, p_1) + \frac{2}{\pi}\int_0^\infty \frac{V_l(p;\, p_2)\, t_l(p_2;\, p_1;\, k^2)\, p_2^2}{k^2 - p_2^2 + i\varepsilon}\, dp_2, \tag{1}$$

where $t_l(p;\, p_1;\, k^2)$ is the partial probability amplitude for the scattering of a wave packet with energy k^2; p and p_1 are the magnitudes of the momenta of the wave packet before and after scattering; and ε is an infinitesimal indicating the rule for going around the contour of integration in the complex p_2 plane.

The kernel of Eq. (1) contains the function

$$V_l(p;\, p_1) = \int_0^\infty j_l(pr)\, V(r)\, j_l(p_1 r)\, r^2 dr, \tag{2}$$

where $j_l(pr)$ is a spherical Bessel function, and $V(r)$ is a function characterizing the steady perturbing field. We consider a $V(r)$ of the form

$$V(r) = V_1(r) + V_2(r), \tag{3}$$

where $V_1(r)$ is the positive function

$$V_1(r) = \begin{cases} V_0^1 \gg k^2, & 0 < r < r_c, \\ 0, & 0 > r_c, \end{cases} \tag{4}$$

and $V_2(r)$ is a negative function of two types:

$$V_2(r) = \begin{cases} -V_0, & r_c < r < r_0, \\ 0, & r > r_0 \end{cases} \tag{5}$$

or

$$V_2(r) = -V_0[1 + \exp((r - r_0)/a)]^{-1}, \quad r_c < r < \infty. \tag{6}$$

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TABLE 1. Numerical Values of Lippmann—Schwinger Equation

$\mathrm{Re}t_0(p;\ p_1;\ k^2)$ $\mathrm{Im}t_0(p;\ p_1;\ k^2)$	[1,1]	[2,2]	[3,3]
$\mathrm{Re}t_0^1$ (2,83; 2; 2)	—0,0506714	—0,0505977	—0,0505409
$\mathrm{Im}t_0^1$ (2,83; 2; 2)	—0,0074178	—0,0074963	—0,0074551
$\mathrm{Re}t_0^1$ (3,16; 2; 2)	—0,0426693	—0,0425717	—0,0425918
$\mathrm{Im}t_0^1$ (3,16; 2; 2)	—0,0064262	—0,0062882	—0,0062783
$\mathrm{Re}t_0^1$ (4,0; 2; 2)	—0,0231618	—0,0231031	—0,0231216
$\mathrm{Im}t_0^1$ (4,0; 2; 2)	—0,0032462	—0,0030588	—0,0030333
$\mathrm{Re}t_0$ (2,6; 0,1; 0,01)	0,0361155	0,0360340	0,0360334
$\mathrm{Im}t_0$ (2,6; 0,1; 0,01)	0,0039234	0,0034471	0,0034464
$\mathrm{Re}t_0$ (3,1; 0,1; 0,01)	0,0814177	0,0808449	0,0808466
$\mathrm{Im}t_0$ (3,1; 0,1; 0,01)	0,0067017	0,0065679	0,0065308
$\mathrm{Re}t_0$ (3,6; 0,1; 0,01)	0,0793683	0,0780568	0,0780590
$\mathrm{Im}t_0$ (3,6; 0,1; 0,01)	0,0060697	0,0061469	0,0061841

Note: $\mathrm{Re}t_0^1(p;\ p_1;\ k^2)$ and $\mathrm{Im}t_0^1(p;\ p_1;\ k^2)$ for function (3) using (5) and the parameters $V_0 = 0.35\ F^{-1}$, $r_0 = 2.54$ F, $r_c = 0.1$ F for an energy of 165.88 MeV, i.e., $k^2 = 4.00\ F^{-2}$; $\mathrm{Re}t_0 \cdot (p;\ p_1;\ k^2)$ and $\mathrm{Im}t_0(p;\ p_1;\ k^2)$ using (6) and the parameters $V_0 = 1.22\ F^{-2}$, $r_0 = 1.564$ F, $r_c = 0.25$ F, $a = 0.3$ F for an energy of 0.415 MeV, i.e., $k^2 = 0.01\ F^{-2}$; [n, n], Padé approximant with numerator and denominator of n-th degree.

Equation (1) is solved numerically for $l = 0$, when the integral (2) for $V_2(r)$ of the type (5) is given by

$$V_0(p;\ p_1) = \frac{V_0^1}{pp_1}\left[\frac{\sin((p_1-p)r_c)}{2(p_1-p)} - \frac{\sin((p_1+p)r_c)}{2(p_1+p)}\right] - \\ - \frac{V_0}{pp_1}\left[\frac{\sin((p_1-p)r_0)}{2(p_1-p)} - \frac{\sin((p_1+p)r_0)}{2(p_1+p)} - \frac{\sin((p_1-p)r_c)}{2(p_1-p)} + \frac{\sin((p_1+p)r_c)}{2(p_1+p)}\right] \tag{7}$$

for $p_1^2 \neq p^2$ and by

$$V_0(p;\ p_1) = \frac{V_0^1}{p^2}\left[\frac{\sin(2pr_c)}{4p} - \frac{r_c}{2}\right] + \frac{V_0}{p^2}\left[\frac{\sin(2pr_0)}{4p} - \frac{r_0}{2} - \frac{\sin(2pr_c)}{4p} + \frac{r_c}{2}\right] \tag{8}$$

for $p_1 = p$. For $V_2(r)$ of the type (6), the integral (2) is given by

$$V_0(p;\ p_1) = V_0^1\int_0^{r_c}\frac{\sin(pr)\sin(p_1r)}{pp_1[1+\exp((r-r_0)/a)]}\,dr - V_0\int_{r_c}^{A}\frac{\sin(pr)\sin(p_1r)}{pp_1[1+\exp((r-r_0)/a)]}\,dr \tag{9}$$

for $A \leqslant (21a + r_0)$. The values of the variable p and the parameters p_1 and k^2 are chosen from the ranges $0.1 \leqslant p \leqslant 6\ F^{-1}$, $0.01 \leqslant k^2 \leqslant 16\ F^{-2}$, and $0.1 \leqslant p_1 \leqslant 4\ F^{-1}$, where $1\ F = 10^{-13}$ cm. The value of V_0^1 is taken as 100 F^{-2}, since for these ranges of p, p_1, and k^2 for $V_1(r) > 100\ F^{-2}$ it practically does not affect the accuracy of the solution of (1) [3].

Equation (1) was solved by the Padé method [3]. The function $t_0(p;\ p_1;\ k^2)$ was taken as the ratio of two n-th degree polynomials:

$$t_0(p;\ p_1;\ k^2;\ g) = P_n(p;\ p_1;\ k^2;\ g)/Q_n(p;\ p_1;\ k^2;\ g) + O(g^{2n+1}), \tag{10}$$

where g is the expansion parameter and $O(g^{2n+1})$ is the remainder term.

Equation (10) is an n-term continued fraction. Since $t_0(p;\ p_1;\ k^2)$ is a complex function, Eq. (10) is also a complex quantity $\mathrm{Re}t_0 + i\mathrm{Im}t_0$. The $\mathrm{Re}t_0(p;\ p_1;\ k^2)$ and $\mathrm{Im}t_0(p;\ p_1;\ k^2)$ were determined in the linear [1, 1], quadratic [2, 2], and cubic [3, 3] approximations.

Table 1 lists the values of $\mathrm{Re}t_0(p;\ p_1;\ k^2)$ and $\mathrm{Im}t_0(p;\ p_1;\ k^2)$ calculated for certain values of p, p_1, and k^2 in the ranges indicated. $\mathrm{Re}t_0(p;\ p_1;\ k^2)$ and $\mathrm{Im}t_0(p;\ p_1;\ k^2)$ behave similarly for other values of the parameters.

Table 1 shows that the [3, 3] approximation ensures a solution which is accurate to 1 part in 10^{-4}. It should be noted that this approximation also gives the same accuracy for a function (3) containing only $V_2(r)$ [3]. Hence, including $V_1(r)$ in the function (3) does not change the order or the approximation in the numerical solution of the Lippmann–Schwinger equation for the range of p, p_1, and k^2 values indicated.

LITERATURE CITED

1. Yu. A. Surinov, Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 7 (1948).
2. L. D. Faddeev, Tr. Mat. Inst. im V. A. Steklova Akad. Nauk SSSR, 69 (1969).
3. M. I. Zhurina, V. N. Novozhilova, and A. M. Popova, Izv. Akad. Nauk SSSR, Ser. Fiz., 39, No. 10 (1975).

DETERMINATION OF THE ELECTRODYNAMIC AND THERMAL FLUCTUATION CHARACTERISTICS OF A BICONICAL CAVITY

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A method is proposed for the design of a biconical cavity with finite wall conductivity. The quality of the resonance volume, the temperature field in its walls, and also the level of natural fluctuational thermal radiation are determined.

A number of papers, which are mainly experimental in nature [1-3], are devoted to irregular limit cavities. Theoretical computations of high-quality oscillation systems of similar nature have also been performed [4]. However, the demands of the practice of accurate measurements by using volume resonance apparatus require a strict method of determining the electrodynamic and noise properties of a biconical cavity, its average temperature over the volume, the thermal coefficient, and the maximum allowable dissipation power. The computation of these characteristics is the aim of this paper.

This problem reduces to the solution of the Maxwell equation (rot = curl)

$$\operatorname{rot}\mathbf{E}(\mathbf{r}) = -ik\mathbf{H}(\mathbf{r}), \quad \operatorname{rot}\mathbf{H}(\mathbf{r}) = ik\varepsilon\mathbf{E}(\mathbf{r}) \tag{1}$$

and heat-conduction equation

$$\Delta T(\mathbf{r}) - \frac{1}{\varkappa} W(\mathbf{r}) = 0. \tag{2}$$

The dissipative function $W(\mathbf{r})$ ($\varkappa$ is the coefficient of thermal conductivity of the cavity wall material) has the form [5]

$$W(\mathbf{r}) = -\frac{c}{8\pi} \operatorname{Re} \operatorname{div} [\mathbf{E}(\mathbf{r}) \times \mathbf{H}(\mathbf{r})]. \tag{3}$$

We determine the thermal and electrodynamic characteristics of the cavity under investigation in the approximation of given thermal sources and temperature, respectively. Such a linearization of the system of equations (1)-(2), which corresponds physically to neglecting the mutual influence of the temperature and the complex conductivity of the cavity walls, is admissible in the following cases: relatively low power level of the working microwave field and weak dependence of the electrodynamic parameters of the cavity wall material on the temperature.

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